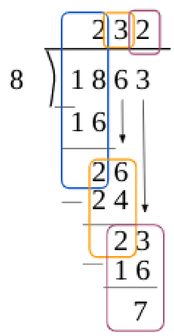
Math 1320: Long and Synthetic Division of Polynomials

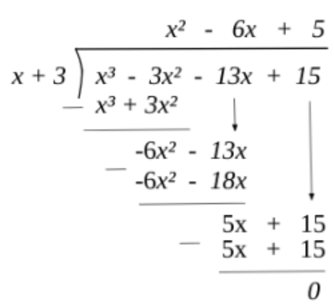
Why is polynomial division important? In this course, we will find zeros of polynomial functions. We have already learned how to find zeros for polynomials of degree 2 by factoring, but how can we find the factors of a larger degree polynomial? We will need to be able to perform polynomial division. We will look at two different types of polynomial division, synthetic and long division. Both processes will achieve the same result, so you may choose which type of polynomial division you like best.

Long Division of Polynomials

Consider the long division of whole numbers:

	**Process:** • 8 goes into 18 twice, with a remainder of 2. We then bring down the next digit in the dividend (6). • 8 goes into 26 three times, with a remainder of 2. We then bring down the next digit in the dividend (3). • 8 goes into 23 twice, with a remainder of 7. There are no more digits in the dividend to bring down.	**Final Answer:** remainder $1863 \div 8 = 232 + \frac{7}{8}$ dividend quotient divisor

Polynomial long division has a similar process, but we have variables, too.

Process of Long Division of Polynomials		**Example**	
Step 1	Arrange the terms of both the dividend and the divisor in descending powers of any variable.	Divide $x^3 - 3x^2 - 13x + 15$ by $x + 3$.	Terms were already arranged in descending powers of x.
Step 2	Divide the first term in the dividend by the first term in the divisor. The result is the first term of the quotient.		Divide x^3 by x to get x^2. This is the first term of quotient.
Step 3	Multiply every term in the divisor by the first term in the quotient. Write the resulting product beneath the dividend with like terms lined up.		Multiplied each term in the divisor $(x + 3)$ by x^2, aligning terms of the product under like terms in the dividend.
Step 4	Subtract the product from the dividend.		Subtract $x^3 + 3x^2$ from $x^3 - 3x^2$ by changing the sign of each term in the lower expression and adding.
Step 5	Bring down the next term in the original dividend and write it next to the remainder to form a new dividend.		Bring down $-13x$ from the original dividend and add algebraically to form a new dividend.
Step 6	Use this new expression as the dividend and repeat the process until the remainder can no longer be divided. This will occur when the degree of the remainder (the highest exponent on a variable in the remainder) is less than the degree of the divisor.	$\frac{x^3-3x^2-13x+15}{x+3} = x^2 - 6x + 5$	Repeat the process, finding the next terms of the quotient.

⋆ Note: Be sure to fill in any missing terms with zeros before performing long division (e.g. $x^4 - 13x^2 + 36 \quad \rightarrow \quad x^4 + 0x^3 - 13x^2 + 0x + 36$)

Synthetic Division of Polynomials

Before using synthetic division, we must first be sure that the divisor is of the form $x - c$.

<table>
<tr><th colspan="2">Process of Synthetic Division of Polynomials</th><th>Example:
Divide $x^3 - 3x^2 - 13x + 15$ by $x + 3$.</th></tr>
<tr><td>Step 1</td><td>Rewrite the divisor in the form $x - c$</td><td>$x + 3 = x - (-3)$</td></tr>
<tr><td>Step 2</td><td>Arrange the polynomial in descending powers, with a 0 coefficient for any missing term.</td><td>Terms are already arranged in descending powers, with no missing terms.</td></tr>
<tr><td>Step 3</td><td>Write c for the divisor, $x - c$. To the right, write the coefficients of the dividend.</td><td>-3 | 1 -3 -13 15</td></tr>
<tr><td>Step 4</td><td>Write the leading coefficient of the dividend on the bottom row.</td><td>-3 | 1 -3 -13 15
↓
1</td></tr>
<tr><td>Step 5</td><td>Multiply c times the value just written on the bottom row. Write the product in the next column in the second row.</td><td>-3 | 1 -3 -13 15
↓ -3
1</td></tr>
<tr><td>Step 6</td><td>Add the values in this new column, writing the sum in the bottom row.</td><td>-3 | 1 -3 -13 15
↓ +-3
1 -6</td></tr>
<tr><td>Step 7</td><td>Repeat this series of multiplications and additions until all columns are filled in.</td><td>-3 | 1 -3 -13 15
↓ +-3 +18 +-15
1 -6 5 0</td></tr>
<tr><td>Step 8</td><td>Use the numbers in the last row to write the quotient, plus the remainder above the divisor. The degree of the first term of the quotient is one less than the degree of the first term of the dividend. The final value in this row is the remainder.</td><td>$\frac{x^3-3x^2-13x+15}{x+3} = x^2 - 6x + 5$</td></tr>
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Practice Problems

Divide using synthetic or long division. Try both methods at least once. State the quotient and remainder.

1. $(x^3 - 2x^2 - 5x + 6) \div (x - 3)$ $[x^2 + x - 2]$
2. $\frac{4x^3-3x^2+3x-1}{x-1}$ $[4x^2 + x + 4 + \frac{3}{x-1}]$
3. $(6x^5 - 2x^3 + 4x^2 - 3x + 1) \div (x - 2)$ $[6x^4 + 12x^3 + 22x^2 + 48x + 93 + \frac{187}{x-2}]$